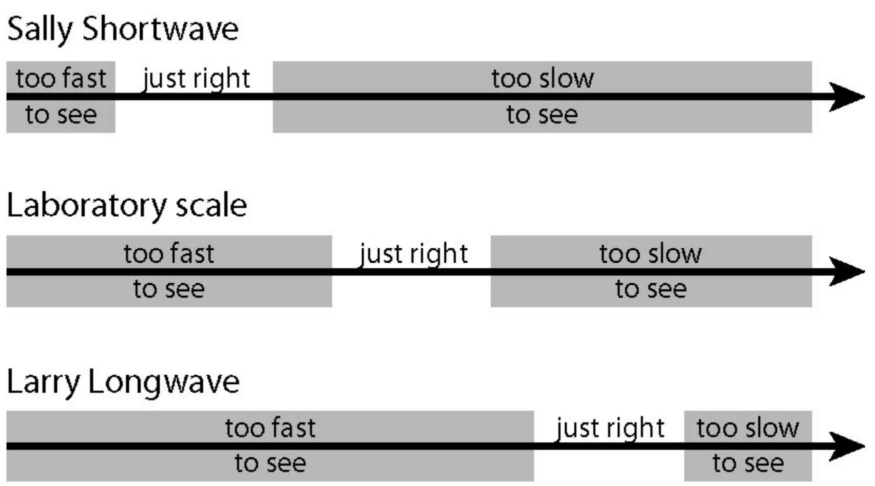
Maxwellian velocity distributions in Slow Time

Christopher Essex — Department of Applied Mathematics, The University of Western Ontario, London, Ontario, Canada, essex@uwo.ca

Bjarne Andresen —Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark, andresen@nbi.ku.dk

Every observer (measuring instrument) has its own timescale:



We do not see the jitter of molecular motion. Our observations average over that (typically Maxwellian) velocity distribution. What would we observe if we observe over a much longer timescale than our own? The pictures illustrate how some effects disappear while others become obvious, previously obscured by transients.

Sun over parking lot

6 mo

Busy campus intersection

1/125 sec

10min

Niagara river

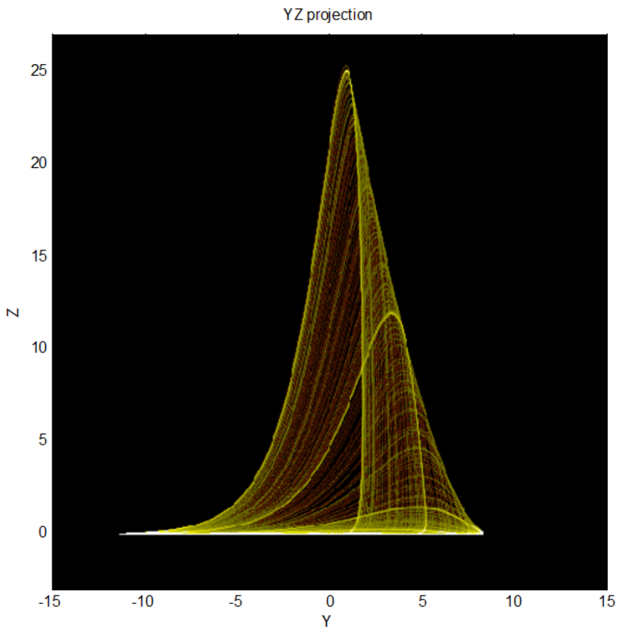
0.4 sec

50 sec

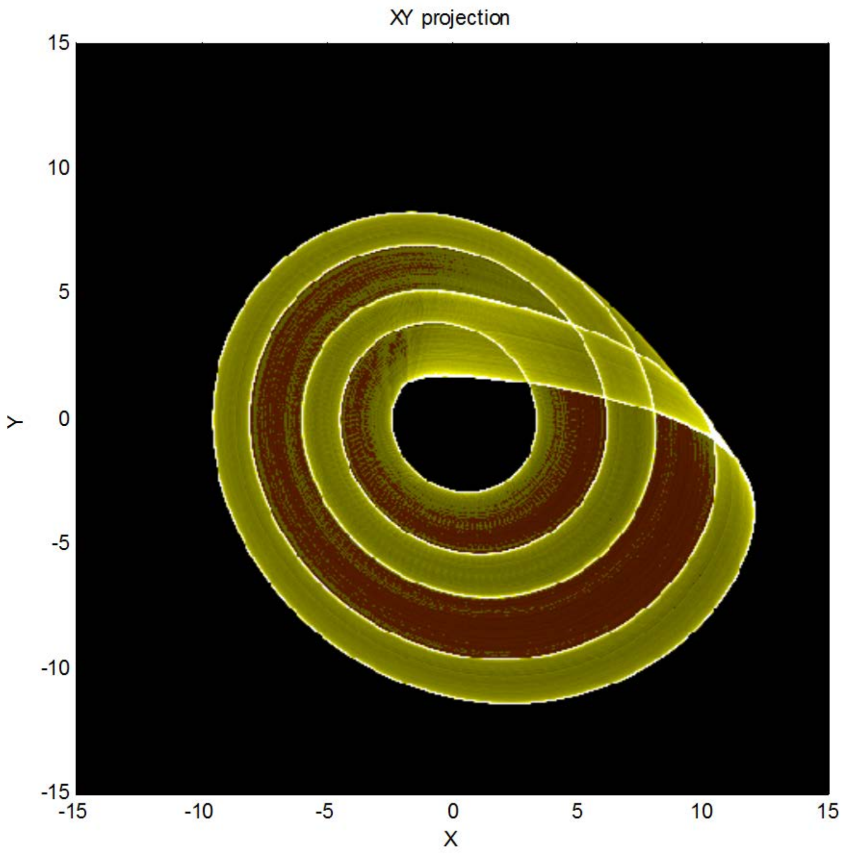
<u>Long-time iterated maps – 10^7 steps</u>

Rösler equations:

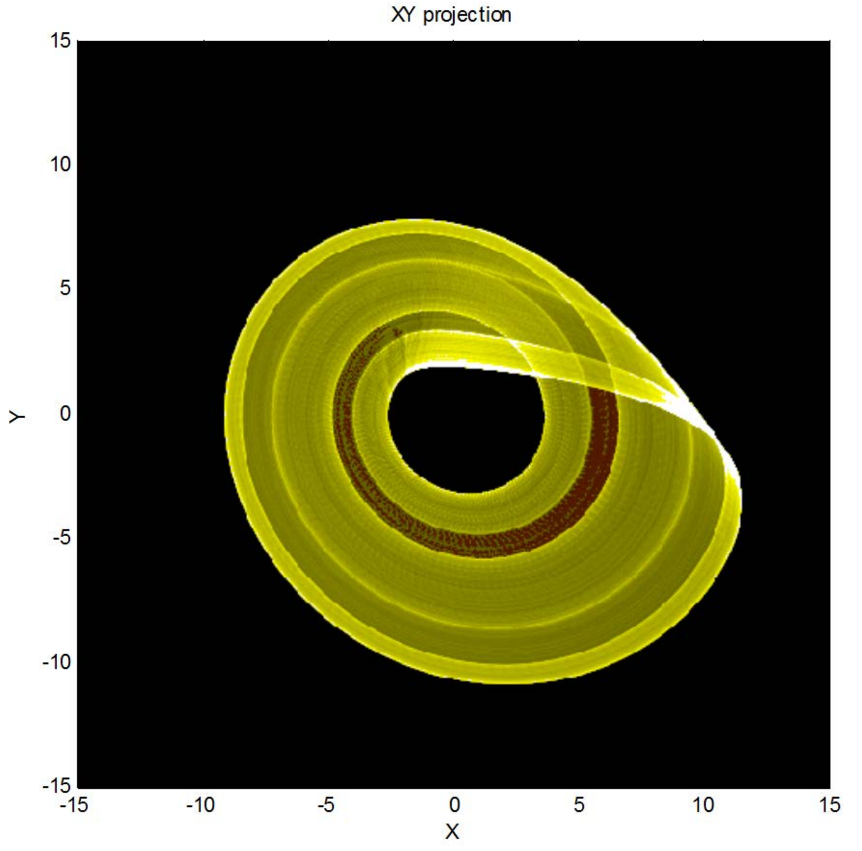
$$\dot{x} = -y - z$$
$$\dot{y} = x + 0.2\,y$$
$$\dot{z} = xz - 5.2\,z + 0.2$$



C=5.981



C=5.982



<u>On the long timescale we observe distributions of distributions.</u>

Mean air velocity (wind) fluctuates and turns into a slightly increased temperature, "thermalizing" wind:

$$p(v;\theta) = \left(\frac{m}{2k\theta}\right)^{1/2} \frac{1}{\sqrt{\pi}}\, e^{-\frac{m}{2k\theta}v^2} \qquad \theta = \frac{\sigma_u^2 m}{k} + T$$

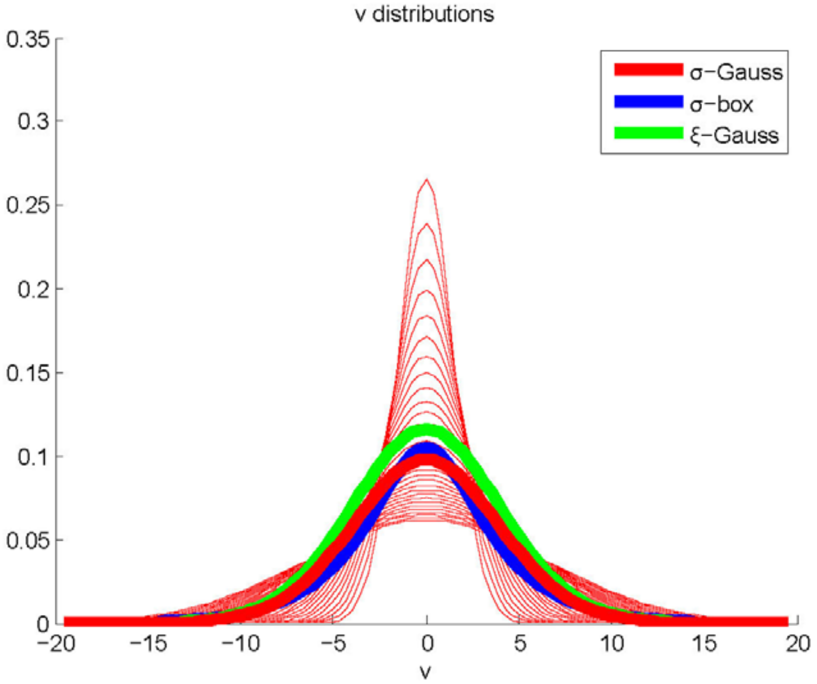
By contrast, temperature fluctuations do not result in a new temperature, a new Maxwellian distribution:

$$p(v;w,\psi_0) = \int_{-\psi_0}^{\infty} p_{v\xi} p_{\xi} d\xi =$$
$$\left[\frac{1+\mathrm{erf}\left(\frac{w^2\psi_0}{(v^2+w^2)^{1/2}}\right)}{1+\mathrm{erf}(w\psi_0)}\right] \frac{w^3\psi_0}{\sqrt{\pi}(v^2+w^2)^{3/2}}\, e^{-\frac{w^2\psi_0^2 v^2}{v^2+w^2}} + \frac{1}{1+\mathrm{erf}(w\psi_0)} \frac{w}{\pi(v^2+w^2)}\, e^{-w^2\psi_0^2}$$

The result is polynomial tails of degree either -3 (infinite fluctuation domain) or -2 (semi-infinite domain).

[*w* is the precision of the velocity precision]

Distribution of standard deviations



Convoluted distribution compared to standard Maxwellian