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REGULAR ARTICLE

Energy landscapes: topographies, interparticle forces and dynamics, and how they are related

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Abstract Many-body systems exhibit dynamic behavior ranging between two extremes that can be characterized as structure-seeking and glass-forming, when allowed to cool to a solid structure from a melt or high-energy random structure. This discussion reviews what has been learned about the origins of these two kinds of behavior, in terms of how long-range forces give rise to relatively smooth energy landscapes, cooperative behavior and structure-seeking character, while short-range interparticle forces do the opposite and induce glass-forming character.

Keywords Energy landscapes · Structure-seekers · Glass-formers · Atomic clusters · Proteins · Kinetics · Master equation

1 Introduction

One of the challenges of molecular and nanoscale science is finding the relationship between the interparticle forces and the extent to which a system becomes a glass or an ordered structure as it cools from the liquid state. A natural approach to address this problem examines the topography of the energy landscape on which the system moves. However, the dimensionality of that landscape grows linearly with the number of particles in the system and, much more significant, the numbers of minima and of saddle points grow very much faster. Hence, one faces a problem of true complexity if one wishes to extract useful physical insights from information about that topography. And that already contains the tacit assumption that only a single potential surface, i.e. a single electronic state in the sense of the Born–Oppenheimer approximation, is involved in the system's behavior.

This problem is far from solved, but considerable progress has given us many insights and indications of where we should be looking to deepen our understanding of this problem. We may even hope to be able to have some control, eventually, over how a system behaves on its complex potential surface, for example by controlling the composition or the external conditions. Here, we review some of the recent advances that have given us some of those insights.

The key phenomenon that is the focus of this work is the recognition that some systems are extremely good at finding their way to special, ordered structures that are a tiny minority of the locally stable forms on a complex potential surface, while others "get lost" very easily, that is, those systems can reach any of a vast number of relatively disordered structures, completely or partially amorphous. The first kind we now call "structure-seekers", and the latter "glass-formers". The natural questions to ask are these: "What characteristics of the potential energy surface distinguish structure-seekers from glass-formers?" "How is the structure-seeking or glass-forming character of a system related to the interparticle forces in the system?" "Is there a way to establish some kind of scale to express the degree to which any chosen system is a structure-seeker or a glass-former?" "Are there other ways, apart from topographical considerations, to recognize distinctions between structure-seekers and glass-formers?" All these

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are questions to which we have at least partial answers, but we cannot yet say we have definitive ways to address them.

2 The archetypal systems

The first system we recognized as what we later called a "structure-seeker" was a cluster of ions $(KCl)_{32}$, whose lowest energy structure is a 4 × 4 × 4 rocksalt cube [1, 2]. This system has a few hundred low-energy rocksalt structures, of which that cube is the lowest. It also has a vast number, somewhere between 10^{12} and 10^{14}, of local minima at which the structures are amorphous, not at all like rocksalt. An example is shown in Fig. 1. It seemed that this would imply that experiments could be done, by rapid cooling of molten clusters of $(KCl)_{32}$, that would produce amorphous alkali halides, which might well be expected to have properties, mechanical and electronic, quite different from those of systems of the same size with rocksalt structures. We carried out simulations to try to capture those amorphous structures, but, to our amazement, according to the simulations, only if the cooling from the liquid state were faster than 10^{13} K/s, corresponding to removing virtually all the energy from the cluster in 5–10 vibrational periods, would it be possible to keep the system from finding a rocksalt structure! This was, of course, quite an amazing result. Not long thereafter, Cheng and Landman carried out a related simulation of depositing fast NaCl clusters on solid argon [3]. This revealed that the impact would melt the alkali halide cluster, and that the rare gas atoms would evaporate so rapidly that they would carry off the energy of the cluster in 10^{-13} s or less and would leave the cluster in an amorphous structure. Hence, the basis for an experiment was laid out clearly—but, unfortunately, it has yet to be carried out.

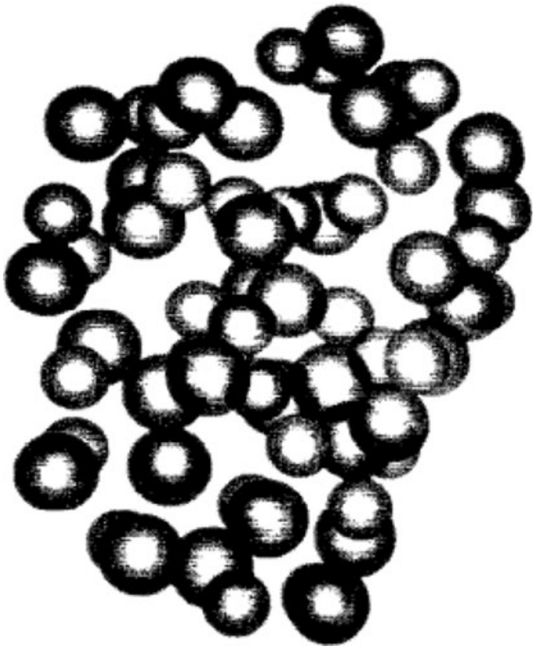

Fig. 1 An amorphous structure of $(KCl)_{32}$

Then, we examined the way rare gas clusters, bound by Lennard-Jones pairwise interactions, would find stable structures when cooled from a molten state. The simulations of this kind of process showed a very different kind of behavior. If, for example, Ar_{19}, with a double icosahedron as its lowest energy geometric structure, is cooled at a rate as slowly as 10^9 K/s, four orders of magnitude slower than that to capture an amorphous alkali halide, no more than half the clusters find their way to that lowest stable minimum. The others are trapped in amorphous structures. This gave the first sharp indication of the distinction between structure-seekers and glass-formers.

Next, we examined the topographies of the energy landscapes on which these systems moved about. We chose as a means to examine the topographies a schematic representation showing the energies of successive geometrically linked stationary points, beginning with a structure at a deep minimum, going up in a random direction to a saddle and then down to a higher minimum. This process was continued so that the energies of the minima increase monotonically in a sequence; we termed these zig-zag patterns "monotonic sequences", so we could see how successive locally stable states could be populated as the system moved lower and lower on its potential surface. The results for the alkali halide cluster and the rare gas cluster were dramatically different and immediately gave us a qualitative basis for the distinction between structure-seekers and glass-formers. The glass-formers have monotonic sequences of stationary points that look like a sawtooth pattern, with very little drop in energy between one locally stable structure and the next. In contrast, structure-seekers have landscapes whose monotonic sequences look like rough staircases, with moderately frequent large drops in energy. Figure 2 shows a comparison of the two cases, taken from the first comparison we made of these [4]. Figure 3 shows schematic diagrams of two energy landscapes, the first of an extreme structure-seeker for which the topography very strongly guides the system to a single deep energy minimum. And the second, a case in which there are at least two deep minima, well separated, so that where the system goes can depend sensitively on its initial state.

One very important physical consideration was immediately apparent in the comparison of these two cases. The potassium chloride cluster was bound by Coulomb forces, represented by a Born–Mayer potential, Coulomb at long range and exponential for the short-range repulsions. The argon cluster was represented by Lennard-Jones attractions

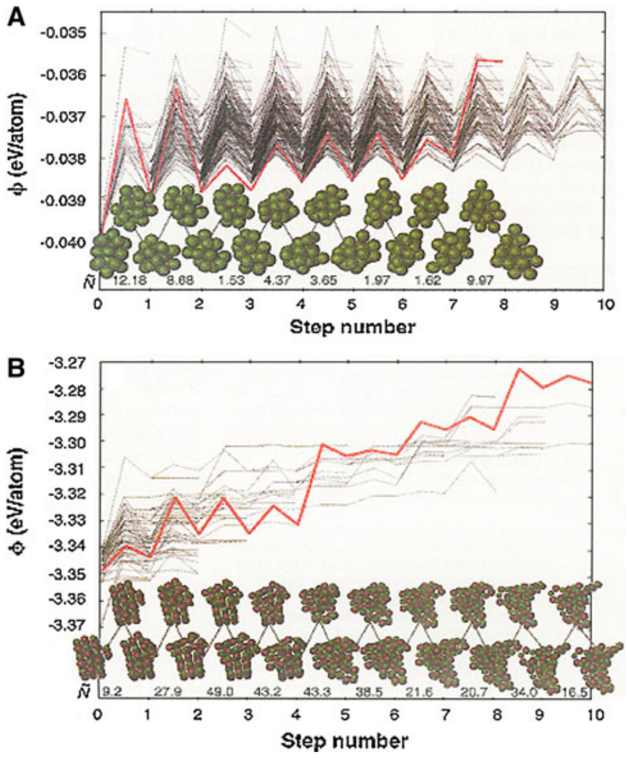

Fig. 2 Monotonic sequences (black zig-zags), with one randomly selected sequence in each set (red zig-zags) and images of the structures at the corresponding points along the sequences. **a** Ar_{19}. which, in the chosen sequence, goes to its global minimum; **b** $(\mathrm{KCl})_{32}$, which, in this chosen sequence, goes to a rocksalt structure but not the global minimum 4 × 4 × 4 cube. Vertical axes show binding energy per atom, in eV

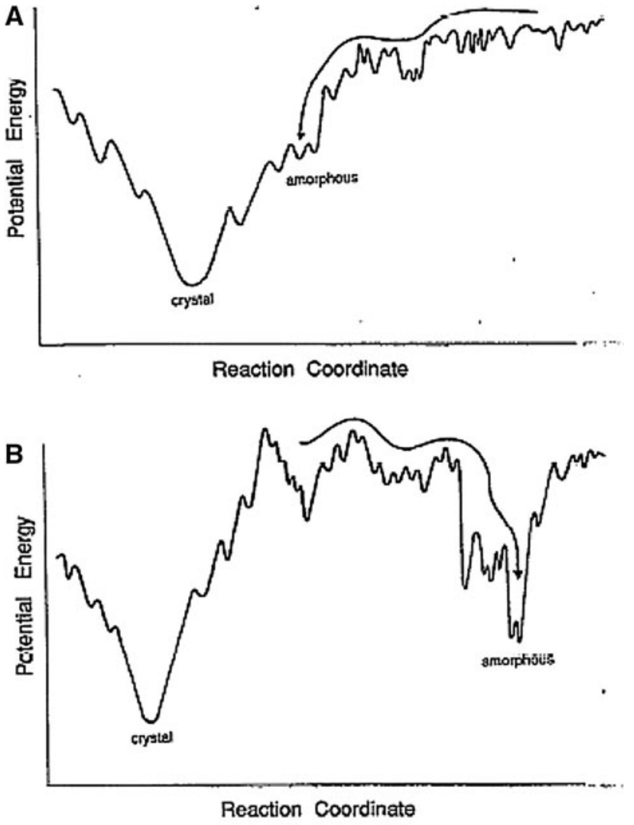

Fig. 3 Two schematic examples of energy landscapes; **a** an extreme structure-seeker's energy landscape; **b** an energy landscape for an intermediate case between an extreme structure-seeker and an extreme glass-former

and repulsions, a short-range force indeed. This difference is apparent from the numbers shown between the successive minima. These are the effective numbers of particles that move when the system goes from one minimum to the next, a quantity based on the one introduced by Stillinger and Weber [5]. We let d_{ij} be the distance between particles i and j. Specifically, the effective number of particles moving is

$$\tilde{N} = N/\gamma$$

where

$$\gamma = \left\langle d_{ij} \right\rangle^4 \Big/ \left\langle d_{ij}^2 \right\rangle^2,$$

with the average taken over these distances, so that $\tilde{N}$ is 1 if only one particle moves, and is N if all N particles move. We see immediately that very few particles move in the argon cluster as it passes between minima, while many particles move in the alkali halide cluster, in its passages between adjacent minima.

Naturally, when the notion of a staircase topography seemed associated with structure-seeking behavior, the question almost immediately arose as to whether foldable proteins also have staircase landscapes. The first case we explored was a simple 46-bead model that had been introduced as a lattice model [6, 7] and was then generalized to a continuous model [8]. This system has some eight or nine deep minima with rather similar structures. We found that the topographies leading to each of these has staircase character, so that the system is structure-seeking insofar as it goes to the deep minima and does not stay in the others [9]. On the other hand, it is not like a protein that folds to a unique region of a potential surface, which is a common supposition. (There is an interesting question of how much variation an organism can tolerate in a protein's structure if those variations are not directly involved in the functioning of the active site of the protein.)

The next step was investigating the potential surface of a real protein. For that purpose, we used bovine pancreatic trypsin inhibitor (BPTI), whose folding we simulated with a simple backbone-bending model [10, 11]. From the forward and backward rates of individual steps, we were able to infer the topography of BPTI, which is indeed extremely staircase-like. We compared that with the topography of a model with the same amino acids as those of BPTI but randomly scrambled, to give a polypeptide that does not

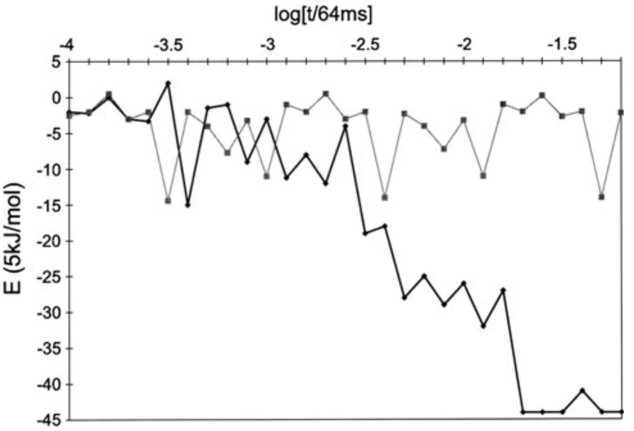

Fig. 4 Topographies of the energy landscape of the protein BPTI (heavy zig-zag) and of a nonfolding, scrambled peptide composed of the same amino acids (light zig-zag)

fold. The effective landscape topographies of these two are shown in Fig. 4. We must infer that the maintenance of the polymer chain, plus effects of contacts between distant residues of the protein, plays the role for this system—and presumably for proteins generally—that the long-range Coulomb potentials play for the alkali halides.

3 Deeper diagnostics and tools of inference

Naturally, one would like to do a number of things when one has the information just discussed. For example, one would like to move to kinetics, to describe quantitatively the behavior of clusters, proteins and nanoscale materials, in terms of their energy landscapes. But the landscapes are far too complex to permit us to even begin to think of writing the full kinetics in terms of the time dependence of the populations of all the local minima on the surface. There are simply far too many of those, and even more of the saddles that connect them. The number of local minima that are geometrically distinct appears from simulations to increase exponentially with N, the number of particles in the system, and each of these has approximately N! permutational isomers for an N-particle system, so the number of minima increases roughly as N!exp(N), possibly even as N!exp(aN) with $a > 1$, one of the fastest increasing functions we know. However, the number of minima increases even faster [12].

Hence, if we want to use kinetics to describe the behavior of these systems, we must rephrase what it is that we want to know. The most important rates are, of course, the slowest, especially for proteins. Hence, we need to find a way to sample the topography of the complex energy landscape in a way that allows us to construct a set of coupled kinetic equations, altogether called the "master equation", which gives as its eigenvalues fairly accurate values for the rate coefficients of the *slow* processes. It seems that no general process for this had been known. Trying several different kinds of selection criteria yielded one that does appear to give a good representation of the slow eigenvalues [13]. This is based on selecting rough sequences that link the deep minima with whatever initial distribution one wishes to use. High-barrier, "bottleneck" pathways dominate the distributions that gave the best approximate representations of the slow eigenvalues that had been found by analyzing full systems. Other criteria were not successful; for example, selecting pathways with low barriers tended to carry the systems into "wrong" regions from which they could not escape using the pathways available in the sample distribution. This analysis was based both on several highly idealized model systems, starting with 100 minima arranged in 10 rows of 10 minima up to one with 37 rows of 70 minima, linked in a simple rectangular pattern, and also on the Ar_{13} cluster. Figure 5 shows the distributions of eigenvalues for the idealized 37 × 70 system, based on "rough pathway" sampling, with successively smaller samples in the successively lower curves. The rate coefficients for the slowest processes are on the right.

Another direction that seems particularly natural as a way of following what we learned in the first part of this work is identifying the relation between the interparticle forces, particularly their ranges, and the character of the topography of the energy landscape. A simple, systematic exploration of that question took the form of examining the distributions of minima and saddles for three closely

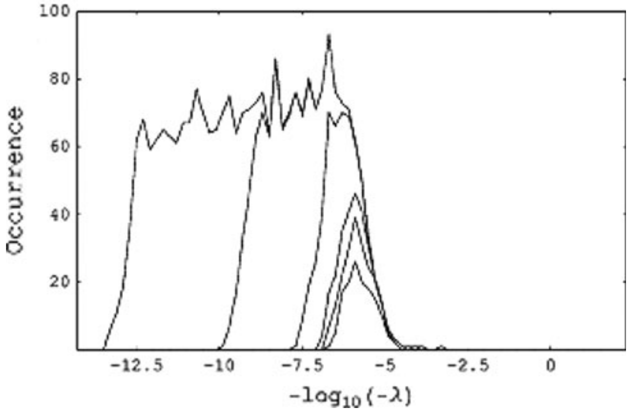

Fig. 5 Eigenvalue distributions for successively smaller samples from the 37 × 70 perturbed landscape, showing that the "rough topography" sampling process is able to reproduce the eigenvalues—the rate coefficients—for the slow processes. A similar result was found for this sampling process with relaxation processes of the Ar_{13} cluster. The leftmost distribution is based on the full simulation; successive distributions are smaller by factors of two

related clusters, namely thirteen particles bound by Morse potentials with range parameters ρ of 4, 5 and 6 [14]. Smaller clusters had been studied previously in a somewhat similar way [15]. The Morse potential is

$$V(r) = \varepsilon\left\{[g(r) - 1]^2 - 1\right\}$$

with

$$g(r) = e^{-\rho(r - r_0)}.$$

The larger is ρ, the shorter is the range of the potential. If $\rho = 6$, the Morse potential most closely approximates the Lennard-Jones potential. Hence, this study probed the Lennard-Jones-like and longer-ranged potentials. Shorter ranges have also been studied, and for very large systems, e.g. for clusters of C_{60} molecules, for which $\rho = 13.6$, a sort of “molecular Velcro” [12]. The known stable diatomic molecules have range parameters that fall between about 3 and 7. A useful way to compare the potentials for these three 13-particle clusters is through examining their “disconnection diagrams” which display the minima and the energies to which one must go to pass from one to another [16–18]. In the diagrams of Fig. 6, the horizontal distance between minima is indeed proportional to the distance in the multidimensional energy space between the minima. This is one extension of the use of disconnection or lid diagrams. We see the obvious evolution of complexity as the range of the pairwise interaction potential changes from 4 to 5 to 6.

We see in these three diagrams a vivid exposition of how the range of interaction between particles governs the degree of complexity of the landscape, and, through that, the way the range of interaction is a determining factor in the extent to which a system is a structure-seeker or a glass-former. The long-range forces, which ensure that movement of any particle influences the motions of other, more distant particles as well as near neighbors, ultimately smooth the landscape and bring systems down to special, select structures. Another way to express this is to say that the topographical entropy of a system with short-range forces is greater than that of a similar system with longer-range forces, and that the behavior of a glass-former is dominated by entropic considerations, while that of a structure-seeker is dominated by enthalpic or energetic factors.

Other characteristics of the potential surface’s topography may seem responsible for structure-seeking or glass-forming character. Distributions of barrier heights and of energies of minima and saddles seem perhaps to be meaningful characterizations, but what seems to be an even more probing kind of information is the distribution of *asymmetries* of the barriers, .e.g. the ratio of the barrier from the higher-energy minimum to the saddle, to the barrier energy on the side of the lower-energy minimum, which we can probe in various ways. One that we can examine here is the distribution of asymmetries as a

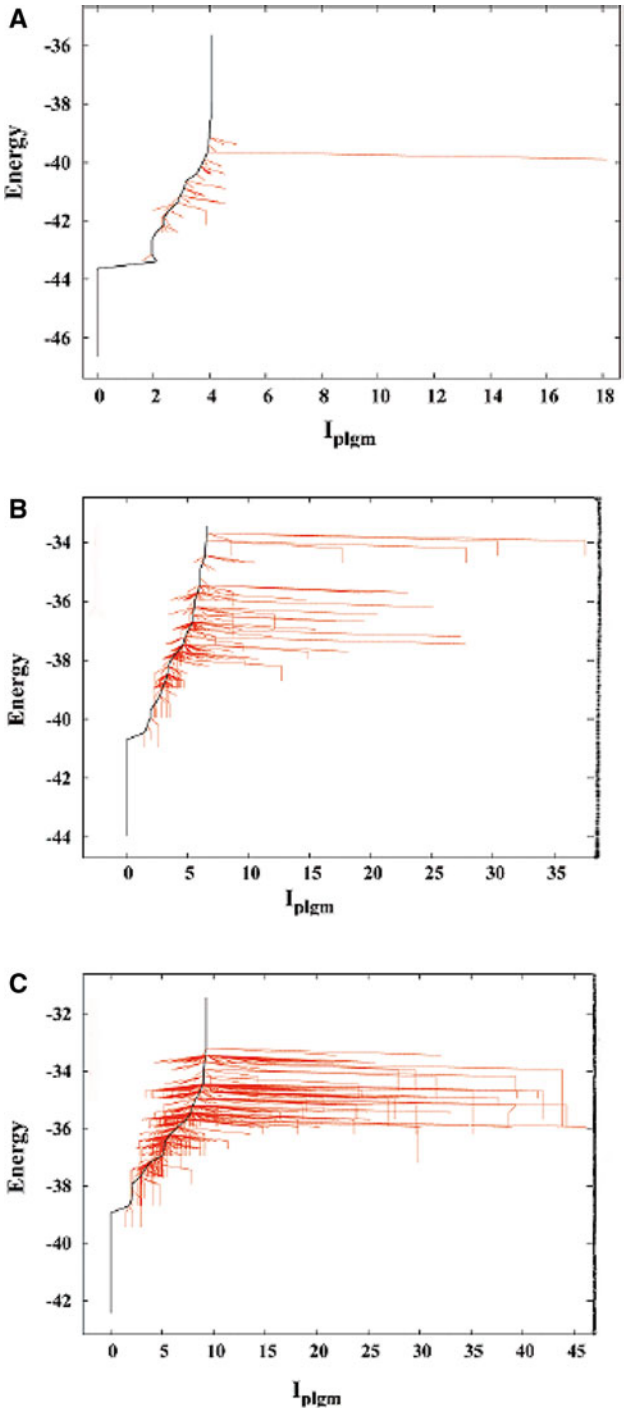

Fig. 6 Disconnection diagrams for 13-particle clusters bound by Morse potentials with range parameters ρ of **a** 4, **b** 5, and **c** 6. The horizontal distances in the diagrams are average integrated path length to the global minimum (I_{plgm}) in terms of the units of the full coordinate space, between the various minima and the global minima, which are shown in black

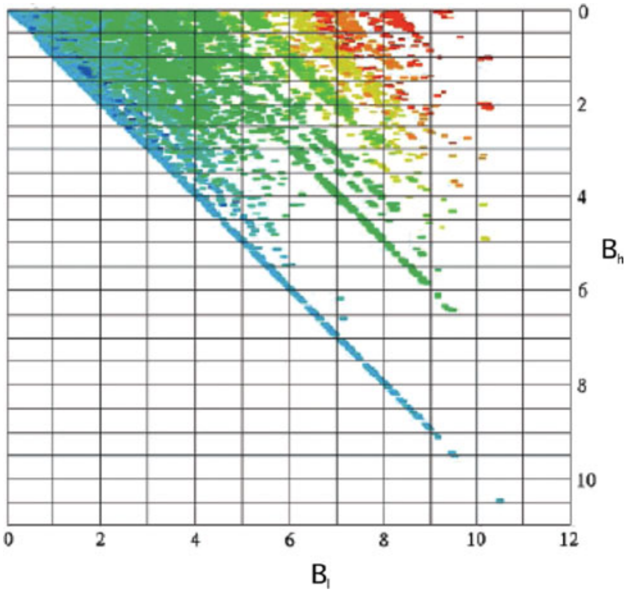

Fig. 7 Barrier energies as functions of the energies of the higher-energy minimum, B_h, and the lower-energy minimum, B_l, and as functions of how deep on the energy surface the energies lie. The blue end of the color scale is the high-energy end, and the red, the deepest. The long, blue diagonal indicates that at high energies, the barriers tend to be symmetric, and the deeper one explores on the surface, the more asymmetric are the barriers. This pattern is that for the range parameter $\rho = 5$

function of how high the energies lie on the surface. This is displayed for the intermediate case, $\rho = 5$, in Fig. 7. This reveals how the barriers at the high-energy parts of the potential surface tend to be symmetric and then become more and more asymmetric as one looks deeper and deeper into the surface, so that the red parts of the diagram are the furthest from the diagonal where B_h and B_l are equal.

4 Future directions

There are clearly a number of questions that remain unanswered in this general area. We still do not really know how the interparticle forces of a protein or a nucleic acid make these such impressive structure-seekers, not at the same level of understanding that we now have of atomic clusters. The fact that there are “hot spot” amino acids whose substitution by mutation spoil the structure-seeking character of a protein tells us that there is richness in the detailed behavior of folding proteins that is far more complex than the crystallization of an alkali halide.

We would like to have some way to quantify the extent to which a system is a structure-seeker or a glass-former, some kind of scale between these two extremes. With that, we would like to be able to relate the scale to the range of the interparticle forces, at least for atomic clusters. In fact, we do have work in progress now, involving Julius Jellinek and Chengju Wang, which is indicating that there is a region in the range of values of the range parameters for various kinds of atomic clusters, in which one can see a transition between dominant structure-seeking character and dominant glass-forming character. This work will be forthcoming soon.

Acknowledgments The author would like to dedicate this paper to the memory of his friend and former colleague Jürgen Hinze. We shared adjacent offices at The University of Chicago for many years, before he moved to Bielefeld. Those were delightful and extremely stimulating times. Although we never wrote a paper together, we discussed many ideas and certainly influenced one another’s thinking. I shall always have vivid and warm memories of him.

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